

A model for anisotropic strange stars

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We attempt to find a singularity free interior solution for a neutral and static stellar model. We consider that (i) the star is made up of anisotropic fluid and (ii) the MIT bag model can be used. The total system is defined by assuming the density profile given by Mak and Harko [1], which satisfies all the physical conditions of a stellar system and is stable by nature. We find that those stellar systems which obey such a non-linear density function must have maximum anisotropy at the surface. We also perform several tests for physical features of the proposed model and show that these are mostly acceptable within certain range. As a special mention, from our investigation we find that the maximum mass and radius of the quark star are $11.811km$ and $3.53M_{\odot}$ respectively.

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I. INTRODUCTION

In the present day context the physical properties and internal structure of the compact stars are topics of active research. Itoh [2] first suggested that the quark stars may exist in hydrostatic equilibrium. According to Bodmer [3] the quark matter made of u , d and s quarks is more stable than any ordinary nuclear matter. Cheng et al. [4] argued that after a supernova explosion a massive star core collapses to a strange star. It was also proposed by Alford [5] that in the dense core of a neutron star there is sufficiently high density and low temperature to crush the hadrons into quark matter.

It is quite interesting that the equation of state (EOS) chosen by the following authors in Refs. [6, 7, 8, 9, 10, 11] for neutron stars did not explain the properties of the observed compact stars like *PSR J1614 – 2230*, *Vela X – 1*, *PSR J1903 + 327*, *Cen X – 3*, *SMC X – 1*, whereas the strange matter EOS can explain them. In the interior of such stars there may exist many exotic phases. However, in the present work our investigation is mostly restricted to the EOS of strange matter only. Rahaman et al. [12] using the MIT bag model proposed a new model for strange stars where they analysed the physical properties of the stars from 6 km to the surface. Here we choose phenomenological MIT bag model [13, 14, 15], according to which, the EOS of strange

matter can be written in a linear form as following

$$P_r = \frac{1}{3}(\rho - 4B_g), \quad (1)$$

where ρ , P_r and B_g are the energy density, the radial pressure and the bag constant respectively. In the bag model the vacuum pressure B_g confirms the quark confinement that equilibrates the pressure of quarks to stabilize the system.

In this paper using MIT bag model we have proposed a new stable model for strange stars, which is neutral, static as well as anisotropic fluid spheres valid under the appropriate physical condition. Using the observed value of the mass of strange stars we have analysed their physical properties and also predicted respective radius of the stars.

We have organized our study as follows: In Sec. II basic equations and the solutions for different physical parameters are provided. We have discussed various physical features of the strange stars in Sec. III whereas provide data sets for the strange stars along with a comparative study in the Sec. IV. The article is concluded with a short discussion in Sec. V.

II. BASIC EQUATIONS AND SOLUTIONS

We consider that the interior of the strange stars is well described by the following space time metric

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (2)$$

where ν and λ are functions of the radial coordinate r only.

The energy-momentum tensor for the proposed strange star model is given by

$$T_0^0 = \rho, \quad (3)$$

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$$T_1^1 = -P_r, \quad (4)$$

$$T_2^2 = T_3^3 = -P_t, \quad (5)$$

where ρ , P_r and P_t in Eqs. (3), (4) and (5) represents the energy density, radial and tangential pressures respectively for the fluid sphere.

The Einstein Field equations are now can be listed below:

$$e^{-\lambda} \left(\frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = 8\pi\rho, \quad (6)$$

$$e^{-\lambda} \left(\frac{1}{r^2} + \frac{\nu'}{r} \right) - \frac{1}{r^2} = 8\pi P_r, \quad (7)$$

$$\frac{1}{2}e^{-\lambda} \left(\frac{1}{2}(\nu')^2 + \nu'' - \frac{1}{2}\lambda'\nu' + \frac{1}{r}(\nu' - \lambda') \right) = 8\pi P_t. \quad (8)$$

Let us define the mass function $m(r)$ of the star as

$$m(r) = 4\pi \int_0^r \rho(r) r^2 dr. \quad (9)$$

Now assuming that the matter density profile inside the quark star can be described as proposed by Mak and Harko [1]

$$\rho(r) = \rho_c \left[1 - \left(1 - \frac{\rho_0}{\rho_c} \right) \frac{r^2}{R^2} \right], \quad (10)$$

where ρ_c and ρ_0 are the central and surface densities of the star of radius R respectively r being the radial coordinate.

Using Eq. (9) and Eq. (10) we find the total mass of the quark star as

$$M = \frac{4}{15} (2\rho_c + 3\rho_0) \pi R^3. \quad (11)$$

At the surface the radial pressure must be zero and hence from Eq. (1) we have

$$\rho_0 = 4B_g. \quad (12)$$

With the choice of EOS (1) and using Eqs. (2), (6) - (10) and (12) we obtain the following physical parameters:

$$\lambda(r) = -\ln \left(1 - \frac{8}{3} \pi r^2 \rho_c + \frac{8}{5} \frac{\pi r^4}{R^2} (\rho_c - \rho_0) \right), \quad (13)$$

$$\nu(r) = \left[\frac{B}{A} \left\{ \operatorname{arctanh} \left(\frac{C}{A} \right) - \operatorname{arctanh} \left(\frac{D}{A} \right) \right\} - \frac{2}{3} \ln (Er^4 + F) + G \right], \quad (14)$$

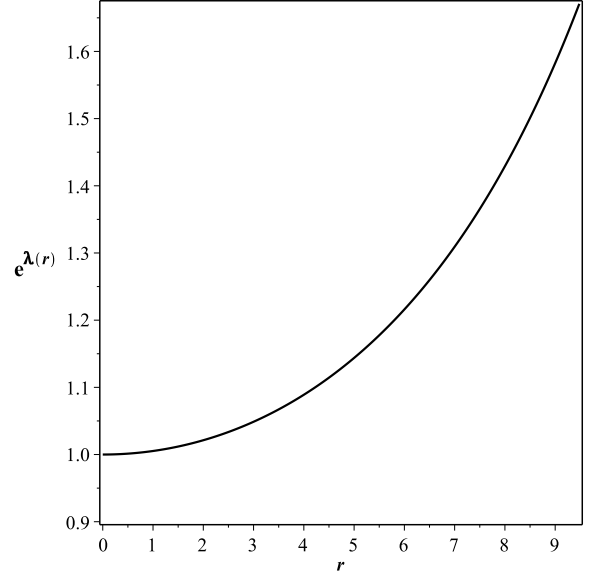


FIG. 1. Variation of $e^{\lambda(r)}$ as a function of the radial coordinate r for the strange star *SMC X-1*

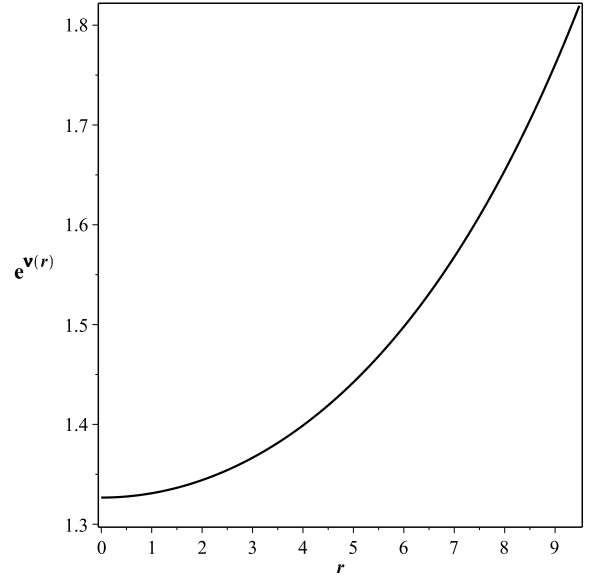


FIG. 2. Variation of $e^{\nu(r)}$ as a function of the radial coordinate r for the strange star *SMC X-1*

$$P_r = \frac{1}{3} (\rho_c - \rho_0) \left[1 - \frac{r^2}{R^2} \right], \quad (15)$$

$$P_t = \frac{c_1(c_2r^4 - c_3) - 16\pi c_1^2r^6 - 10c_4r^2}{120R^2 \left[\pi r^2 \rho_c R^2 - \frac{3}{5}c_1\pi r^4 - \frac{3}{8}R^2 \right]}, \quad (16)$$

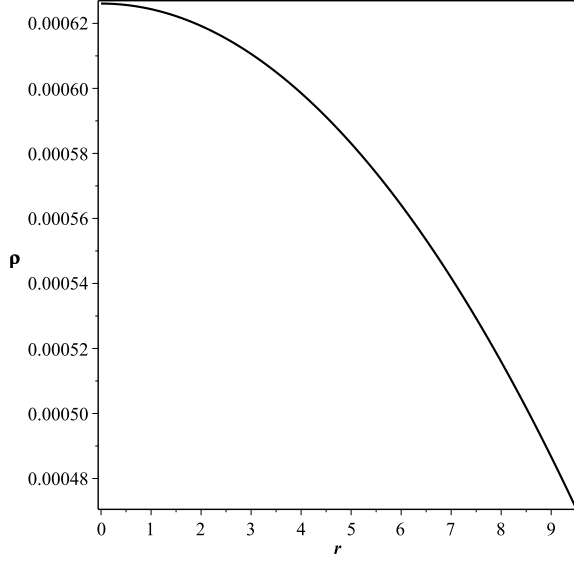


FIG. 3. Variation of the density as a function of the radial coordinate r for the strange star *SMC X-1*

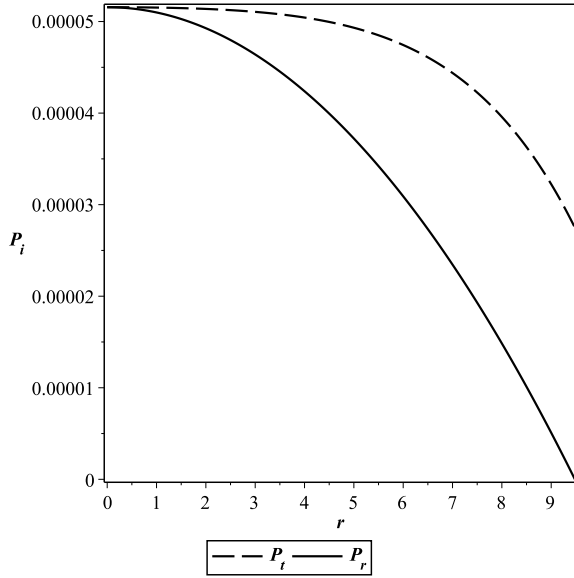


FIG. 4. Variation of the radial pressure and transverse pressure as a function of the radial coordinate r for the strange star *SMC X-1*

$$\Delta(r) = P_t - P_r = \frac{[(12 c_1^2 r^4 - \frac{1}{5} c_1 c_5 r^2 + c_6) \pi - \frac{9}{2} c_1 R^2] r^2}{36 R^2 [r^2 (\frac{3}{5} c_1 r^2 - \rho_c R^2) \pi + \frac{3}{8} R^2]}, \quad (17)$$

where

$$\begin{aligned} A &= \sqrt{\pi^2 R^2 (10 \pi \rho_c^2 R^2 - 9(\rho_c + \rho_0))}, \\ B &= \sqrt{10 \pi R^2 (\rho_0 - 2/3 \rho_c)}, \\ C &= \sqrt{10 \frac{\pi}{5} R^2 (-\rho_c + 6 \rho_0)}, \\ D &= \sqrt{10 \frac{\pi}{5} (-6 r^2 \rho_c + 6 r^2 \rho_0 + 5 \rho_c R^2)}, \end{aligned}$$

$$\begin{aligned} E &= \pi (-24 \rho_c + 24 \rho_0), \\ F &= \pi (40 r^2 \rho_c - 15) R^2, \\ G &= \frac{2}{3} \ln [-15 R^2 + (16 \rho_c + 24 \rho_0) \pi R^4] \\ &\quad - \frac{16}{15} R^2 (\frac{2}{3} \rho_c + \rho_0) \pi + \frac{2}{3}, \\ c_1 &= (\rho_c - \rho_0), c_2 = 32 (\frac{5}{4} \rho_c - \rho_0) \pi R^4, c_3 = 15 R^4, \\ c_4 &= 10 [\pi (4 \rho_c^2 - 2 \rho_c \rho_0 + \rho_0^2) R^2 - 3(\rho_c - \rho_0)] R^2, \\ c_5 &= \frac{1}{5} (156 \rho_c - 84 \rho_0) R^2, \\ c_6 &= (6 \rho_c - 3 \rho_0) (4 \rho_c - \rho_0) R^4. \end{aligned}$$

The behaviour of the above physical parameters are shown in FIGS. 1 - 5 which are quite satisfactory in their nature.

Now let us maximize the anisotropic stress at the surface ($r = R$) without assuming the nature of extremum. We obtain from Eq. (17) as follows

$$\rho_c = - \left[\frac{-28 \pi R^2 \rho_0 + 16 R^4 \pi^2 \rho_0^2 - 15}{32 R^2 \pi (2 \pi R^2 \rho_0 + 1)} \pm \frac{\sqrt{-4304 R^4 \pi^2 \rho_0^2 - 3200 R^6 \pi^3 \rho_0^3 - 120 \pi R^2 \rho_0 + 6400 R^8 \pi^4 \rho_0^4 + 225}}{32 R^2 \pi (2 \pi R^2 \rho_0 + 1)} \right]. \quad (18)$$

The above second order differential equation (17) related to anisotropy takes the form on the surface of the system (i.e. at $r = R$) as follows:

$$\begin{aligned} \Delta''(R) &= - \frac{10}{3 R^2 (24 \pi R^4 \rho_0 + 16 \pi R^4 \rho_c - 15 R^2)^3} [8064 \pi^3 \rho_0^4 R^{12} \\ &\quad + 7296 \pi^3 \rho_0^3 R^{12} \rho_c - 20992 \pi^3 R^{12} \rho_0^2 \rho_c^2 - 2560 \pi^3 R^{12} \rho_0 \rho_c^3 \\ &\quad + 8192 \pi^3 \rho_c^4 R^{12} - 26496 \pi^2 R^{10} \rho_0^3 + 18768 \pi^2 R^{10} \rho_c \rho_0^2 \\ &\quad + 36192 \pi^2 R^{10} \rho_c^2 \rho_0 - 17664 \pi^2 \rho_c^3 R^{10} + 25290 \pi \rho_0^2 R^8 \\ &\quad - 34020 \pi R^8 \rho_0 \rho_c + 10080 \pi \rho_c^2 R^8 + 675 R^6 \rho_0 - 675 \rho_c R^6]. \end{aligned} \quad (19)$$

Now using Eqs. (11), (12) and (18) and taking the value of bag constant as $83 \text{ MeV}/(fm)^3$ [12], we get several solutions for R . However, we examine that among all these solutions only one solution is physically acceptable and also consistent with the Buchdahl condition [16].

III. SOME PHYSICAL FEATURES OF THE MODEL

In this section we shall discuss different physical parameters of the model.

A. The generalized Tolman-Oppenheimer-Volkoff equations

According to Tolman [17], Oppenheimer and Volkoff [18], the sum of forces should be equal to zero so that the system is subjected to the equilibrium,

$$F_g + F_h + F_a = 0, \quad (20)$$

where F_g , F_h and F_a are the gravitational, hydrostatic and anisotropic forces respectively.

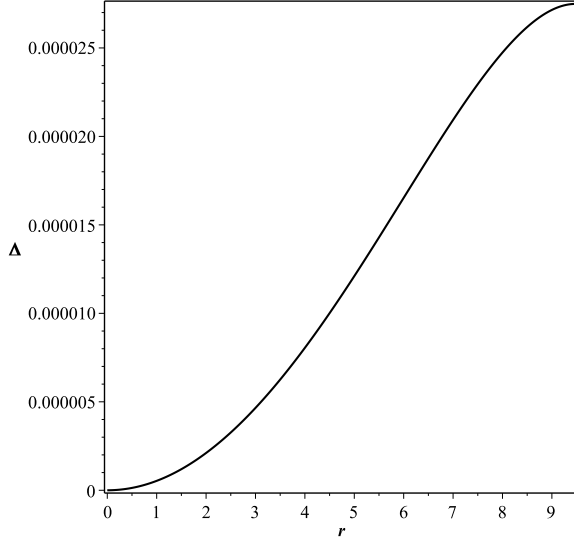


FIG. 5. Variation of anisotropy as a function of the radial coordinate r for the strange star *SMC X - 1*

The generalized TOV equation [19, 20] in its explicit form is

$$-\frac{M_g(\rho + p_r)}{r^2}e^{\frac{\lambda-\gamma}{2}} - \frac{dp_r}{dr} + \frac{2}{r}(p_t - p_r) = 0, \quad (21)$$

where M_g is the the effective gravitational mass of the system whcih is defined as

$$M_g = \frac{1}{2}r^2 e^{\frac{\nu-\lambda}{2}} \nu', \quad (22)$$

where for our system $F_g = -\frac{1}{2}[\rho + P_r]\nu_r$, $F_h = -\frac{d}{dr}P_r(r)$, and $F_a = 2\frac{(P_t - P_r)}{r}$.

We have drawn plot for the above TOV equation in FIG. 6. From this it is clear that our proposed model has achieved equilibrium state under the combined effect of the forces.

B. Energy conditions of the system

To satisfy the energy conditions i.e null energy condition (NEC), weak energy condition (WEC) and strong energy condition (SEC) for an anisotropic fluid sphere composed of strange matter the following inequalities have to be hold simultaneously:

$$NEC : \rho + p_r \geq 0, \rho + p_t \geq 0, \quad (23)$$

$$WEC : \rho + p_r \geq 0, \rho \geq 0, \rho + p_t \geq 0, \quad (24)$$

$$SEC : \rho + p_r \geq 0, \rho + p_r + 2p_t \geq 0. \quad (25)$$

The above energy conditions as drawn in FIG. 7 shows that our proposed model for the strange stars satisfies all the energy conditions.

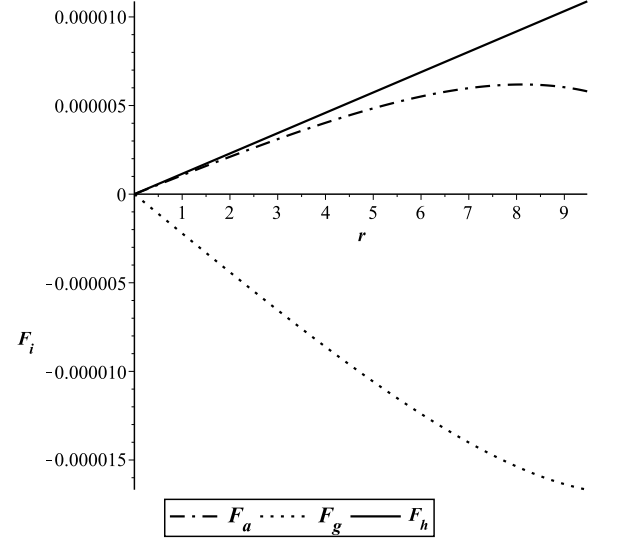


FIG. 6. Variation of different forces as a function of the radial coordinate r for the strange star *SMC X - 1*

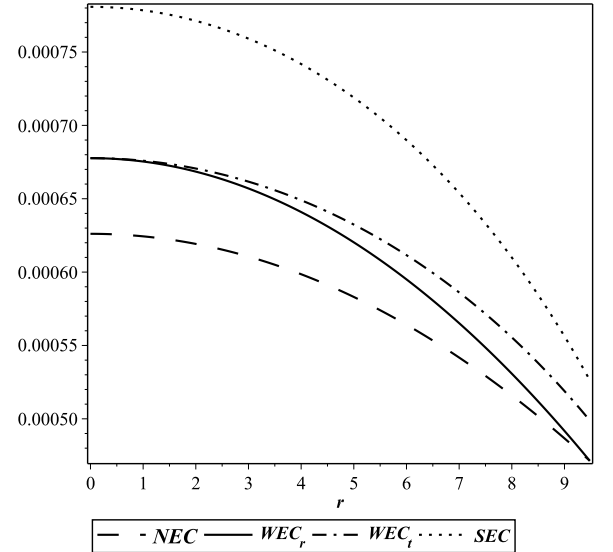


FIG. 7. Variation of different energy conditions as a function of the radial coordinate r for the strange star *SMC X - 1*

C. Stability of the system

Using the concept of Herrera's cracking (or overturning) [21] we shall examine the stability of our system. The condition of causality establishes the physical acceptability of a fluid distribution, which demands $0 \leq v_{st}^2 \leq 1$ and $0 \leq v_{sr}^2 \leq 1$. Also according to Herrera [21] and Andréasson [22] for the stability of the matter distribution another condition required is $|v_{st}^2 - v_{sr}^2| \leq 1$ and this suggests that no cracking i.e potentially stable region must be there. From FIG. 8 we observe that our system satisfies all of these conditions and thus provides a stable

system.

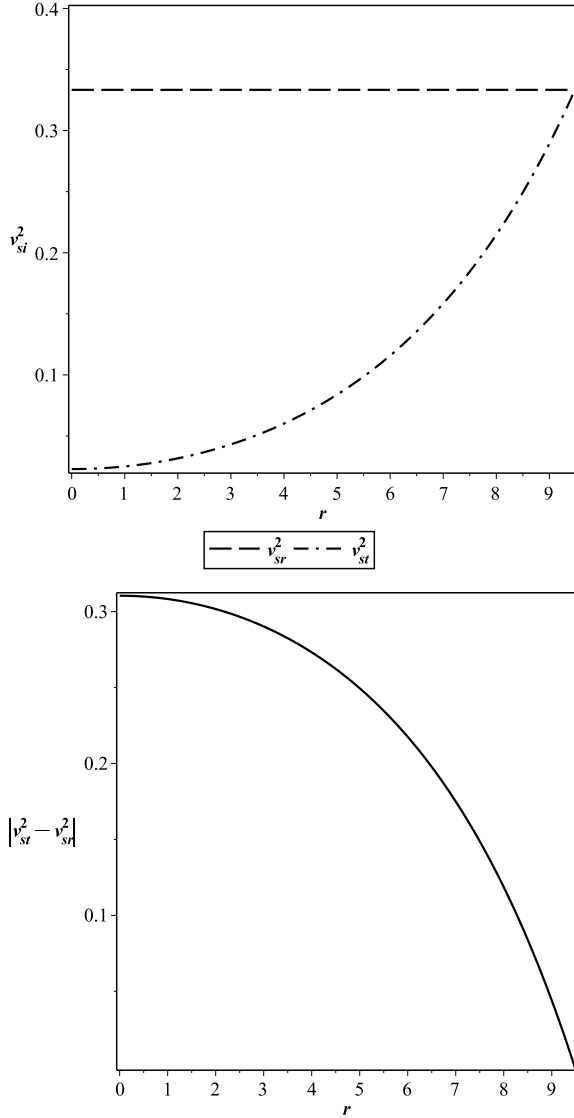


FIG. 8. Variation of the square of sound speed (in the top panel) and modulus of difference of the square of sound speed (in the bottom panel) as a function of the radial coordinate r for the strange star $SMC\ X - 1$

D. Mass-radius relation

The maximum allowable mass-radius ratio for the above proposed anisotropic fluid sphere can be calculated following Buchdahl [16]. He has proposed that the maximum limit of mass-radius ratio for static spherically symmetric perfect fluid star should satisfy the following upper bound $2M/R < 8/9$ ($= 0.88$). However, Mak and Harko [23] have given the generalized expression for the same mass-radius ratio.

The total mass of the anisotropic compact star which

is defined in Eq. (11) can be considered as the maximum mass $M_{max} = 4(2\rho_c + 3\rho_0)\pi R^3/15$. From this mass-radius relation we find out the values of different compact star candidates in TABLE II and observe that all the values fall within the acceptable range as specified by Buchdahl [16]. The maximum mass and radius feature is shown in FIG. 9.

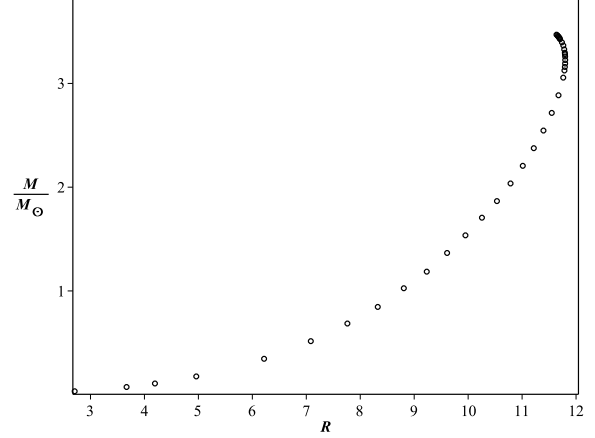


FIG. 9. Mass vs Radius curve for strange stars for $B_g = 83\text{MeV}/(fm)^3$

E. Surface redshift

The compactification factor (u) for the proposed strange star model can be written as

$$u(r) = \frac{m(r)}{r} = \frac{4\pi r^2 (5\rho_c R^2 - 3r^2\rho_c + 3r^2\rho_0)}{15R^2}. \quad (26)$$

So the corresponding surface redshift (Z) as we get from the compactness becomes

$$Z = [1 - 2u(R)]^{-\frac{1}{2}} - 1 = \frac{1}{\sqrt{1 - 8\pi R^2 (\frac{2}{15}\rho_c + \frac{1}{5}\rho_0)}} - 1. \quad (27)$$

The variation of the compactness and redshift with the radial distance are shown in the FIG. 10.

IV. DISCUSSIONS AND CONCLUSIONS

In this paper we have derived radius of the different strange star candidates using their observational mass and bag constant (as $83\text{ MeV}/(fm)^3$), shown in TABLE I. From FIGS. 1 - 3 it is clear that our solution is free from geometrical and physical singularities. Also it can be observed that the metric potentials $e^{\lambda(r)}$ and $e^{\nu(r)}$ have finite positive values in the range $0 \leq r \leq R$. The values of the central density, central pressure, Buchdahl condition and surface redshift for the different strange star candidates of TABLE I are shown in TABLE II.

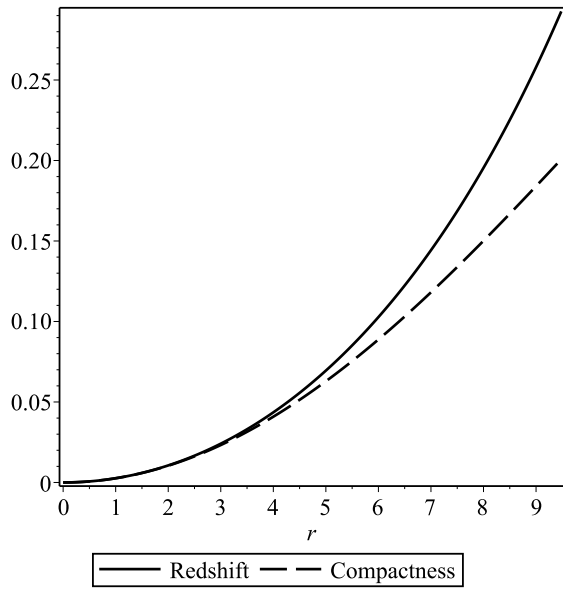


FIG. 10. Variation of the compactness and redshift as a function of the radial coordinate r for the strange star $SMC X-1$

TABLE I. Physical parameters of the observed strange stars

Strange Stars	Mass (in M_\odot)	Mass (in km)	Radius (in km)
<i>PSR J1614 - 2230</i>	1.97 ± 0.04	2.9057 ± 0.059	10.71
<i>Vela X - 1</i>	1.77 ± 0.08	2.6107 ± 0.118	10.39
<i>PSR J1903 + 327</i>	1.667 ± 0.021	2.4588 ± 0.03	10.22
<i>Cen X - 3</i>	1.49 ± 0.08	2.1977 ± 0.118	9.89
<i>SMC X - 1</i>	1.29 ± 0.05	1.9027 ± 0.073	9.48

It is shown from our model that the surface density of the star $SMC X-1$ is $6.36 \times 10^{14} \text{ gm/cm}^3$ which is very high and consistent with ultra compact stars [24, 25, 26]. We also have found out that the redshift of the stars in TABLE II are within the range (0.29 – 0.48) which is consistent under the constraints $0 < Z \leq 1$ and $Z \leq 2$ as suggested by the investigators respectively in the following Refs. [27, 28, 29, 30, 31] and [16, 32, 33].

From Eq. (17) and FIG. 5 one can find that the anisotropic force $F_a = 2(P_t - P_r)/r$ is positive throughout the system, i.e. $P_t > P_r$ and hence the direction of the anisotropic force is outward for our system.

The equation $\frac{dR}{d\rho_c} = 0$ suggests that the star has maximum value of radius R_{max} for $\rho_c|_{R_{max}}$. From the equation $\frac{dM}{d\rho} = 0$, the star has maximum value of mass for $\rho_c|_{M_{max}}$. The model yields the values of the maximum mass and maximum central density as $M_{max} = 3.53M_\odot$ and $\rho_c|_{M_{max}} = 2 \times 10^{15} \text{ g/cm}^3$ respectively.

Interestingly, in the present study we find that the anisotropy of the compact star is zero at the center and then it is increasing through out the interior region of the star and reach it's maximum value at the surface. Again from Eqn. (19) it is clear that the $\Delta''(R)$ is nega-

TABLE II. Physical parameters as derived from the proposed model

Strange Stars	ρ_c (in gm/cm^3)	P_r (in dyne/cm^2)	$\frac{2M}{R}$	Z
<i>PSR J1614 - 2230</i>	9.54×10^{14}	9.51×10^{34}	0.54	0.48
<i>Vela X - 1</i>	9.18×10^{14}	8.43×10^{34}	0.50	0.42
<i>PSR J1903 + 327</i>	9.01×10^{14}	7.94×10^{34}	0.48	0.39
<i>Cen X - 3</i>	8.73×10^{14}	7.11×10^{34}	0.44	0.34
<i>SMC X - 1</i>	8.44×10^{14}	6.24×10^{34}	0.40	0.29

TABLE III. Physical parameters as derived from the proposed model

Strange Stars	$\Delta(0)$ (in km^{-2})	$\Delta(R)$ (in km^{-2})	$\Delta''(R)$ (in km^{-2})
<i>PSR J1614 - 2230</i>	0	0.000062	-0.0000067
<i>Vela X - 1</i>	0	0.000049	-0.0000054
<i>PSR J1903 + 327</i>	0	0.000044	-0.0000048
<i>Cen X - 3</i>	0	0.000035	-0.0000034
<i>SMC X - 1</i>	0	0.000027	-0.0000031

tive, which supports mathematically that the anisotropy is also maximum on the surface of an anisotropic compact star. As a comparative study, the maximum anisotropy of few quark stars are given in Table III (the values are calculated from our model).

We would also like to mention that the maximum allowable mass to radius ratio is 0.40 as evident from TABLE II for $SMC X-1$. TABLE II also indicates that all the other values, including this value, lie within the acceptable range as predicted by Buchdahl [16], i.e. $2M/R \leq 8/9$.

Hence, as a final comment, from the above discussions it can be concluded that the model proposed in this work seems suitable to study the ultra-dense compact strange stars.

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